COMPUTER-ORIENTED MATHEMATICS FOR A DEVELOPING COUNTRY

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This paper considers how a computer-oriented mathematics curriculum might be introduced into a developing country whose resources might be limited and whose students might not be as well prepared as students from industrialized countries. A specific case study is presented of an institution in a developing country which introduced a computer-oriented mathematics curriculum in conjunction with the government's social action program and the local industry. The performance of computer-oriented mathematics students versus the traditional students is quantitatively evaluated.

1. INTRODUCTION

Education in developing countries has always tended to follow the patterns of education of the former colonial masters. This is particularly true in the case of mathematics where colleges and universities in the provincial capitals usually follow a sophisticated mathematics curriculum which has little relevance to the "masses" and which contributes little, if at all, to the overall development effort of the country.

Many developing countries whose populations are soaring and whose food supply is perennially insufficient produce an annual crop of research oriented mathematicians. This, in itself, is by no means harmful. However, research mathematics does require a "research atmosphere," i.e. colleagues, communication facilities, library facilities and perhaps some hardware. These things are usually lacking in a developing country. As a result the annual crop of research oriented mathematicians is divided into three groups:

(a) Those (usually the "cream of the crop") who become part of the tragic "brain drain" which afflicts so many developing countries.

(b) Those who go into non-related jobs in industry, or

(c) Those who go into to teaching to train <u>more</u> research oriented mathematicians who then either:

  (a) Become part of the "brain drain"

  (b) Go into non-related jobs

  (c) Go into teaching to train more mathematicians who:

    (a) "Brain drain"

    (b) Non-related jobs

    (c) Teach, who then:

      (a)

      (b)

      (c)

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      .
      .

This infinite sequence does little to contribute towards national development.

We have asked ourselves what "kind" of mathematics is required for a developing country without compromising the integrity of the mathematician. This paper documents the efforts of one institution in one developing country seeking an answer to this question in one area of mathematics.

2. THE HOST COUNTRY

Since the "host country" for this project is the Republic of the Philippines a few words of background material might be in order.

The Philippines is an archipelago of 7,107 islands stretching for nearly 2000 kilometers off the southern coast of mainland Asia. As a developing country, the Philippines is faced by a number of serious problems. Agricultural production has been insufficient to meet the existing and growing demands of the country, in spite of the fact that 85% of the population depends on agriculture for their livelihood. Industrialization has not developed sufficiently to provide a firm economic base, nor to effectively utilize the best human and natural resources of the country. Moreover, the industry which has developed is concentrated in only a few areas.

Educationally the Philippines presents a paradoxical picture. Four hundred years of exposure to the "elitist" education of the Spanish colonizers followed by fifty years of exposure to the American concept of universal education produced an educational system which boasts of a schoolhouse in every barrio; one of the highest literacy rates in all of Asia; and for which the per capita enrolment in higher education is second only to the United States (followed by Israel, Australia, Japan and Sweden; Source: UNESCO Statistical Yearbook, 1968).

This paradox, i.e. educational alignment with the developed countries and economic alignment with the developing countries, has produced a poor match between the "output" of the educational system and the "input" for industrial development. The infinite sequence discussed in §1. is most visible in the Philippines.

3. COMPUTER-ORIENTED MATHEMATICS

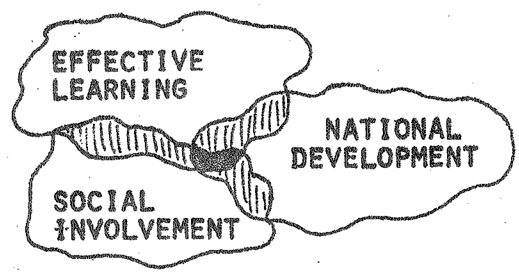
An experimental program in computer-oriented mathematics was introduced at De La Salle College, Manila, during the academic years 1973-74 and 1974-75 to attempt to improve the "match" between academic output and developmental needs in three ways.

First and foremost it was felt that some drastic measures had to be taken to improve the "survival" of students in the mathematics program. In general, students come to college with a poor background in basic mathematical skills. Often times students are graduates of provincial high schools where little mathematics is taught. Students needed to learn mathematics more effectively and it was felt that integrating basic computer programming into the Freshman mathematics curriculum would help develop the psychological schema necessary for effective mathematics learning. (§4. below gives some examples of how this can be accomplished.)

Second, the Philippines, which is only in the beginning stages of industrialization, could swing towards "hard" industry, with their incumbent pollution and environmental problems or towards "soft" industry for which the educational and sociological conditions of the country are better suited. Since there is already a large demand for "computer trained" people in local industry it seemed wise to begin to produce graduates who are capable of assuming positions in this needed area. Furthermore, as the economic problems of developing countries become more complicated and computer simulation techniques become more sophisticated, we can expect that computer trained graduates will be in ever increasing demand; even a cursory knowledge of computer techniques will be a valuable asset. The current, and expected, sharp decline in the cost of hardware can only accelerate this phenomenon. Hence, in the interest of National Development it seemed necessary to introduce students to "the computer" as part of the standard curriculum.

Finally, in a country for which seventy-five percent of the population subsists at the marginal level and for which nearly all of the young people from the upper classes attend college, some sort of social involvement on the part of college students is essential. Accomplishing this as part of the computer-oriented mathematics curriculum would further enhance the value of this curriculum for National Development.

In summary then, the computer-oriented mathematics curriculum at De La Salle College was envisioned to lie in the intersection of the areas of (1) effective learning, (2) National Development, and (3) social involvement, as illustrated below:



4. THE CURRICULUM

The experimental curriculum was divided into three areas: a mathematics area which included the technical materials per se, a National Development area in conjunction with local industry and a social involvement area in cooperation with the government's Youth Civic Action Program (YCAP).

4.1. The Mathematics Area

The standard mathematics curriculum for technically oriented students at De La Salle College consisted of four mathematics courses, each lasting one semester and each worth five credit hours. The four courses were:

Semester I -- Algebra/Trigonometry
Semester II -- Calculus I
Semester III -- Calculus II
Semester IV -- Calculus III

which covered the fundamentals of single and multi-dimensional calculus with the necessary prerequisite algebra, trigonometry and analytic geometry. A one semester "computer fundamentals" course was a corequisite for the second and third semesters.

In the experimental curriculum, the same four semesters were used to cover the same material, but the number of units per course was increased to six. The extra unit counting as a "computer laboratory" for the "computer-oriented" lecture.

In developing the experimental curriculum it was felt that the curriculum must be truly "computer-oriented," not merely the addition of computer related topics to the conventional curriculum. This was to enable the students to become better problem solvers, an aim consistent with our National Development goals. More than just pragmatic considerations, we believe that "perhaps the greatest value to be gained from the study of mathematics is the ability to solve problems," to quote George Pólya {9}. Furthermore, human beings being basically "goal directed organisms" {9}, this would make mathematics "fun."

When we say "problem solvers" we do not mean to restrict ourselves only to the class of trivial problems which the student might encounter in a basic mathematics course; we are trying to establish a "way of life" for the student. Quoting another well known mathematics educator, George Forsythe, "The most valuable acquisitions in a scientific or technical education are the general-purpose mental tools which remain serviceable for a lifetime. I rate natural language and mathematics as the most important of these tools, and computer science as a third" {6}.

In formulating the experimental curriculum arguments were not presented against "computer science" but rather whether a mathematics course was the "right place" for the introduction of such material. Quoting Rosser {8}, "If a mathematics course not only teaches technical proficiency in the computer use of algorithms, but also teaches mathematical techniques which can enable the student to use the algorithms more effectively (or even dispense with them on occasion), but most of all gets the student in the habit of using these mathematical techniques regularly, then without question this is the best place for the student to learn the algorithms."

In addition to the "oriented" aspect, the experimental curriculum uses the computer as a learning tool, quite independent from the computer related topics. The most distinguished American mathematician, Garrett Birkhoff, has said, regarding the use of the computer in basic mathematics courses: "To my mind, the use of computers is analogous to the use of logarithm tables, tables of integrals, carefully drawn graphs of the trigonometric functions, or carefully drawn figures of the conic sections. Far from muddying the limpid waters of clear mathematical thinking, they make them more transparent by filtering out most of the messy drudgery which would otherwise accompany the working out of specific illustrations." {1}

In brief then, the mathematics area of the experimental curriculum aims at integrating problem solving techniques into the basic curriculum, using the computer and to use the computer as a learning tool. The basic topics in the experimental curriculum are:

I. Classical Logic
In this topic we introduce the student to the "mathematical way of thinking." Flow charting is introduced as a way of making procedures "precise."

II. Classical Set Theory
Using the "classical logic" of the previous section we introduce the concept of a set. Flow charting leads naturally into writing simple programs which compute sets of numbers.

At the end of this section the student should have a good understanding of abstract sets, as illustrated by sets of numbers and should be able to write a simple program to tabulate a set such as the set {2,4,6,...}.

III. Number Systems
Using the logic and set theory of the first two sections the Natural Number System is defined as a set of sets, e.g. $1 = \{\phi\}$, $2 = \{1,\phi\}$, etc. Binary operations of + and × are defined on this set by certain rules.

This is the most difficult topic to date for the students. The computer is now used as a learning tool by utilizing some short programs to demonstrate what is meant by closure, commutativity, etc.

Students can now "code" an equation such as 3X - 10 = 5 into a computer program and determine its solution by trial and error.

If the above equation were changed to 3X - 10 = 2, then a solution could not be found within the set of Natural Numbers. This motivates us to look for a number system which could provide a solution for equations like 3X - 10 = 2. The student appreciates that quest for a new number system begins with the problem of determining solutions for equations; his work with some simple equation programs leads to this appreciation.

With this practical problem as motivation, some formal definitions are given for the Integer Number System as was the case with the Natural Numbers.

When the student sees that other equations, such as 3X = 1, cannot be solved within the Integers motivation is provided for the "discovery" of the Rational Number System. In another use of the computer as a learning tool the student demonstrates that every rational number has a repeating decimal expansion. The question is asked "if we construct a decimal expansion which does not repeat, what kind of a number would that be?" and this is shown to be equivalent to asking for solutions to equations such as $X^2 = 2$. Hence, the Real Number System is born.

Finally, attempting to solve equations such as $X^2 = -1$ leads us to the Complex Number System and the student learns how to work with complex numbers in the computer by dealing with pairs of real numbers.

At this point the student has a good deal of familiarity with the basic number systems, from a practical point-of-view and is able to write a variety of simple computer programs. We close this topic by introducing the student to such concepts as "binary operation," "group," "ring," "field," etc. as generalizations of the operations and number systems which they are already familiar with. Using the computer as a learning tool the student constructs tables for arbitrary binary operations, writes a program for "clock addition" and "clock multiplication" and for other number systems which might not be commutative or associative.

IV. Functions

With the established concept of set and a familiarity with the various number systems the student can establish a relationship between one set of numbers and another set of numbers by means of a "rule" or "formula." The use of the computer enables the student to explore such relationships in much greater depth and much more quickly than would otherwise be possible.

Simple relationships such as the pitch/length relationship for a guitar $\{p = 12\ LOG_2(24/\ell)\}$ or the distance/time relationship for a falling body $\{d = \frac{1}{2}gt^2\}$ may be computer plotted and examined in much greater detail than could be done by "hand plotting." When the trigonometric functions and their inverses are introduced as special kinds of "rules and formulas" the computer is used to demonstrate how these functions behave.

The computers real worth is shown when students begin to combine functions by means of "operations" such as addition, multiplication or composition. The student can easily graph a function such as

$$f(x) = \frac{LOG(ASIN(2x^2 + 3x - 7))}{x}$$

and learn that there are some values for which f(x) cannot be computed and that there are restrictions on what values f(x) may have. Hence the concepts of domain and range of a function is established.

The introduction of the traditional "ordered pairs" definition of a function is then merely a generalization of concepts firmly established in the mind of the student and hence is more readily accepted.

V. Limits

Now that students realize that there are points at which functions may not be defined we ask them to consider the function

$$f(\theta) = \frac{\text{SIN}\,\theta}{\theta}$$

which they readily accept as not being defined for $\theta = 0$. Using the computer the student finds that $f(\theta)$ can be made as close to 1 as he likes by making θ sufficiently small. The student appreciates that θ can never be equal to zero and $f(\theta)$ can never assume the value one. We then write

$$\lim_{\theta \to 0} \frac{\text{SIN}\,\theta}{\theta} = 1$$

which has a good deal of intuitive feeling for the student. When we present then the traditional δ-ε definition of the limit students are able to assimilate it as a generalization of their intuitive definition.

The study of limits is one area in which the computer-oriented approach is particularly successful.

VI. Differential Calculus
As with limits, the opportunities for computer applications in differential calculus are numerous. Determining the equation for a secant line through two points on the graph of a function and then using the computer to "see" what happens when the points become "very close" is a natural way to introduce the concept of the derivative of a function.

For example, long before we proved that $D_\theta \text{SIN}\,\theta = \text{COS}\,\theta$ students had derived this relationship by point-by-point differentiation of the Sine function and graphing the results using a computer plotting program.

VII. Integral Calculus
The conventional technique of slicing the "area under the curve" into rectangular strips is used to introduce the definite integral. Using trapezoidal and parabolic strips leads to other integration techniques and is a motivation for discussing errors inherent in the computing process.

A second approach to determining the "area under the curve" by means to random points in the plane (i.e. Monte-Carlo techniques) leads to a discussion of various topics in probability and statistics.

VIII. Other Topics
The above topics are completed during the first year of the experimental curriculum. The second year of the experimental curriculum goes into multi-dimensional calculus and various topics in advanced calculus. Most of these topics are generalizations or extensions of what was covered during the first year.

For example, partial derivatives are now computed as limits of two-dimensional functions and their interpretation in three-dimensions is more general, but the numerical techniques are a common factor which the student recognizes as an extension of his earlier work.

Infinite series is another ideal application for the computer-oriented curriculum. Plotting the individual terms of a Fourier Series expansion for a function and seeing how they "sum up" to the given function is a most rewarding experience for the student.

4.2 The National Development Area

At the conclusion of the first year of the experimental curriculum, students were skilled enough in basic computer programming to assume positions in industry. In cooperation with a number of computer installations, arrangements were made to field students as "trainees" in these computer centers.

As of this writing only one training period has taken place, but the initial results have been most encouraging. Students find the experience a valuable compliment to their classroom work and those in industry are encouraged to find that students are being trained in the field of computer science.

4.3 The Social Involvement Area

Computers can play an important role in National Development and can involve students in work related to "the masses." This was done by establishing a program in conjunction with the National Computer Center and the Youth Civic Action Program of the Department of Education and Culture wherein students volunteer for six week assignments with an agency of the National Computer Center.

Students have worked in the computer centers of, for example, the Department of Agriculture and Natural Resources, the Bureau of Internal Revenue, the Armed Forces of the Philippines Logistics Center and other computer centers where computers are used for social development.

5. RESULTS

Subjectively the student response was most rewarding. As had been experienced in schools abroad, students showed high motivation and nearly all students felt that the computer aided them in better understanding the concepts involved in Calculus. {3}, {11}.

Objectively we were in a quandary as to what type of testing instrument might be utilized. Certainly the second and third aspects of the curriculum could not be evaluated by any testing instrument -- their effects, if any, would take years to measure and would have to be measured on a National scale.

If we subscribe to the "behavioral objectives" philosophy to evaluate purely the mathematical ability of the students then we would have to establish an absolute standard by which to judge the performance of the students {5}. We were not prepared to do that as we were more interested in how the students under the experimental curriculum compared with the the students under the standard curriculum.

We selected as our testing instrument the "standardized" CALCULUS test published by Educational Testing Service {Princeton, New Jersey, USA}. Since the number of students under the experimental curriculum was quite small, each student was tested. Obtaining testing subjects from the standard curriculum was a different matter since participation was on a purely voluntary basis. Therefore samples are not random and statistical techniques could not be applied to the results. In general, however, students who volunteered from the standard curriculum were the better students who felt that they would like to measure their performance on such a standardized test.

The results, stated in the form of %-ile of the average score obtained, were as follows:

TWO SEMESTERS OF STANDARD:	18%-ile
FOUR SEMESTERS OF STANDARD:	25%-ile
TWO SEMESTERS OF EXPERIMENTAL:	70%-ile

That is, students who completed two semesters of the experimental curriculum performed, on the average, above 70% of the American students used as the "control group" for the standardized test.

6. CONCLUSION

As we said, this is but one small effort by one institution in one developing country. Never-the-less we feel that we have demonstrated, at least on a small scale, that there is a place for "computer-oriented" mathematics in a developing country and that it can make significant improvements in both the performance of students and the relevance of mathematics to the problems faced by a developing country.

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